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Finite volume modelling of a stratified flow with the presence of submerged weirs

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The saltwater intrusion in the estuary of the Adige River has been investigated by a two-dimensional finite volume shock-capturing model. Owing to the relative small tide range characterizing the river mouth, a sharply stratified salt wedge tends to form during low discharge periods (e.g. in summer). Suitable hydraulic relations have been introduced to model the action of a submerged barrage, located close to the estuary mouth and built to hinder seawater intrusion. Field measurements of salinity profiles have been used to calibrate the model. The numerical results suggest that, as a consequence of increased water withdrawal that occurred in the last years, the barrage does not prevent efficiently the intrusion of the salt wedge any more.

Keywords: stratified flows and density currents; two-dimensional models; river hydraulics; open channel flow; numerical simulations; estuaries

1. Introduction

Seawater intrusion into the Adige River estuary is a phenomenon known since the early 1970s (Autorità di Bacino del Fiume Adige 2008). The flow field, owing to the relatively low amplitude characterizing the tidal forcing at the river mouth, is usually sharply stratified, with a formation of a well-defined salt wedge moving along the river estuary. The saltwater intrusion has consequences both on the local environment and on human activities (Oude Essink 2001). Withdrawals from rivers are widespread in the Po Valley, and they are carried out to supply water to aqueducts and to irrigate the extensive rural areas surrounding the rivers, dominated by intensive agriculture (Autorità di Bacino del Fiume Po 2006). However, aquifers may be affected by salt pollution, causing a drying up of lands and migration of the local fauna. The aqueducts of Chioggia and Rosolina (touristic beach-cities of the Adriatic Sea) withdraw freshwater a few kilometres upstream of the Adige estuary; therefore confining the salt wedge is of primary importance.

In the 1980s, the Italian Ministry of Agriculture suggested that the construction of a submerged barrage would have helped to preserve the river from seawater intrusion (Autorità di Bacino del Fiume Adige 2008). The barrage was built in 1995 by Consorzio di Bonifica Delta Po Adige (now Consorzio di Bonifica Delta del Po), that is, the competent local government agency, about 4 km upstream of the mouth.

In recent years, however, the barrage has not been working efficiently. Withdrawals increased significantly due to the growth of urbanized land and intensive agriculture. With this growth, the upstream freshwater discharge frequently attains values lower than that needed for the optimal functioning of the barrage. As a consequence, seawater overpasses the barrage and propagates upstream. The possibility that the seawater intrudes upstream of the barrier is clearly higher during summer periods, when freshwater withdrawals increase mostly because of agricultural purposes. Several studies have been carried out by Consorzio di Bonifica Delta del Po and a project to improve the barrage structures has been financed by the government (press release nr. 823/2012 of Regione Veneto government).

Both theoretical and experimental analyses have been conducted to study the effects of an obstacle immersed into a stratified flow field (Armi 1986; Lawrence 1993; Wessels & Hutter 1996; Klymak et al. 2012). Stability analysis has been used to study the behaviour of the interface (Defina et al. 1999; Castro et al. 2007; Liu et al. 2012) and mixing between layers (Balmforth et al. 1998; Holland et al. 2002). Yih and Guha (1954) and Su (1976) carried out an analytical investigation of hydraulic jumps in stratified flows validated by experimental results. The dynamics of internal hydraulic jumps was also investigated by Wood and Simpson (1984) who linked it to mixing and interface shear stress.

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The aim of the present work is to study numerically the salt wedge intrusion in the Adige River. We apply a finite volume shock-capturing model describing the dynamics of a stratified flow and develop a numerical procedure to manage special local elements in the domain, such as those related to the crested barrage.

Section 2 summarizes the physiographic and hydraulic characteristics of the Adige River. Section 3 reports a description of the numerical model that has been implemented to simulate the dynamics of the stratified flow field in the Adige estuary. Section 4 describes how to deal with weirs and spillways in a stratified flow field under unsteady flow conditions. Section 5 reports the results of the application of the numerical model to the final reach of the Adige River, located near the mouth and extending about 9 km upstream of the saltwater barrage. Finally, Section 6 reports the conclusions and some possible future developments.

2. Characterization of the Adige River

The Adige River is the second longest Italian river, with a length of 409 km (Autorità di Bacino del Fiume Adige 2008). The river source is located 1586 m above mean sea level, near the Reschen Pass, close to the borders between Italy, Switzerland and Austria (Figure 1).

The flow path can be divided into three distinct parts. The upstream reach, about 100 km long, flows in the Venosta Valley with a torrential behaviour due to the relatively high bed slope, larger than 0.05 in some sections. The middle reach flows through the Adige Valley for about 200 km, from Bolzano to Verona. The last reach is characterized by a very low slope (in the order of 0.0002) and flows through the Po Valley up to the northern Adriatic Sea. The river bed downstream the city of Verona attains an elevation higher than the surrounding floodplains. A system of levees was therefore built over the years. The hydrological catchment upstream of Verona is about 12,100 km² and is characterized by an alpine climate. The river discharge is fed by frequent precipitations during autumn and winter periods and also by snow melting during spring. Conversely, the river discharge decreases significantly during the summer period.

The astronomical tidal amplitude characterizing the northern Adriatic Sea where the river debouches is about 1 m and is semidiurnal (Figure 2(b)). The flow field in the Adige River estuary is usually strongly stratified owing to the relatively low tidal regime. A salt-wedge intrusion then takes place. The sharply stratified structure of the density distribution has been confirmed by a series of field measures of salinity profiles carried out at several locations by Consorzio di Bonifica Delta del Po in August 2012.

![Figure 1. Geographic characterization of the catchment of the Adige River. The river flows through the cities of Bolzano, Trento and Verona before debouching in the Adriatic Sea.](image)

![Figure 2. Hydraulic conditions measured at the Adige River mouth. Figure (a) shows the measured freshwater discharge (solid line) and the assigned discharge (dashed line) at the upstream section of the computational domain for the numerical simulations. Figure (b) shows the tidal elevations of the Adriatic Sea measured at the Adige mouth and assigned at the downstream boundary of the computational domain.](image)
Figure 3. Salinity profile measured just upstream (a) and downstream (b) of the barrage. The solid line refers to a measurement made during high tide conditions and dashed line to a measurement made in low tide conditions.

Figure 4. Aerial photos of the final reach of the Adige River. The yellow arrow points to the position of the submerged barrage built to contrast the seawater intrusion. UTM coordinates of barrage middle point are 287107 m E, 5002459 m N, 33 T. Source: Regione Veneto archive, http://circe.iuav.it.

Figure 3(a) and 3(b) show two examples of these profiles, measured just upstream and downstream of the barrage for both low tide and high tide conditions. The behaviour of the salt wedge is unsteady, because of the astronomical tide, topographic variations of the river bed and fluctuations in the river discharge.

The above mentioned barrage, built to hinder seawater intrusion, is located about 4.2 km upstream of the Adige River mouth (Figure 4). The structure consists of seven rectangular iron gates measuring $13.00 \times 6.00$ m, with a thickness of 0.40 m, which have openable flaps to allow heavy solid transport and can be extracted when maintenance is needed. Six gates have the upper edge at an elevation $-0.75$ m with respect to mean sea level, whereas the crest of the central gate is $-1.60$ m with respect to mean sea level in order to allow river navigation.

The combined effects of relatively high freshwater discharge and of the barrage should confine the salt water downstream of the structure. The threshold value of freshwater discharge ensuring the optimal operation of the barrage is $80$ m$^3$/s. However, freshwater discharge attains values lower than this threshold value, especially during the summer because of upstream withdrawals. Figure 2(a) shows the temporal trend of the freshwater discharge during the measures of salinity profiles.
3. Numerical model for flow field

The stratified flow field near the river mouth can be modelled by two superposed layers with a sharp interface surface. The lower layer (denoted by index 2) carries seawater whereas the upper one (denoted by index 1) is composed by freshwater. Mean seawater density of the northern Adriatic Sea is \( \rho_2 = 1033 \text{ kg/m}^3 \). Saltwater intrusion is larger during summer periods, when rains are not very frequent and the river discharge is relatively low (Figure 2(a)). Under these conditions solid transport may be neglected, and the density of the upstream coming freshwater can be assumed to be about \( \rho_1 = 1000 \text{ kg/m}^3 \). The density ratio is \( r_\rho = \rho_1/\rho_2 = 0.968 \). The density of each layer is assumed to keep constant in space and time; in other words as a first approximation, we neglect flow exchanges due to mixing between the two layers.

The mathematical model consists of the two-layer two-dimensional shallow water equations governing the motion of two immiscible Newtonian fluids with different densities (Canestrelli & Toro 2012). The equations read:

\[
\begin{align*}
\frac{\partial H_1}{\partial t} + \frac{\partial}{\partial x} \left( q_{1x} + q_{2x} \right) + \frac{\partial}{\partial y} \left( q_{1y} + q_{2y} \right) &= 0 \quad (1a) \\
\frac{\partial q_{1x}}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{q_{1x}^2}{H_1 - H_2} + \frac{1}{2} g (H_1 - H_2)^2 \right] + \frac{\partial}{\partial y} \left( q_{1x} \frac{q_{1y}}{H_1 - H_2} \right) + g (H_1 - H_2) \frac{\partial H_2}{\partial y} &= - \frac{\tau_{int,x}}{\rho_1} \quad (1b) \\
\frac{\partial q_{1y}}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q_{1x} q_{1y}}{H_1 - H_2} \right) + \frac{\partial}{\partial y} \left[ \frac{q_{1x}^2}{H_1 - H_2} + \frac{1}{2} g (H_1 - H_2)^2 \right] + g (H_1 - H_2) \frac{\partial H_2}{\partial y} &= - \frac{\tau_{int,y}}{\rho_1} \quad (1c) \\
\frac{\partial H_2}{\partial t} + \frac{\partial q_{2x}}{\partial x} + \frac{\partial q_{2y}}{\partial y} &= 0 \quad (1d) \\
\frac{\partial q_{2x}}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{q_{2x}^2}{H_2 - b} + \frac{1}{2} g (H_2 - b)^2 \right] + \frac{\partial}{\partial y} \left( q_{2x} q_{2y} \right) + g (H_2 - b) \frac{\partial b}{\partial x} &= \frac{\tau_{int,x}}{\rho_2} - \frac{\tau_{b,x}}{\rho_2} \quad (1e) \\
\frac{\partial q_{2y}}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q_{2x} q_{2y}}{H_2 - b} \right) + \frac{\partial}{\partial y} \left[ \frac{q_{2y}^2}{H_2 - b} + \frac{1}{2} g (H_2 - b)^2 \right] + g (H_2 - b) \frac{\partial b}{\partial y} &= \frac{\tau_{int,y}}{\rho_2} - \frac{\tau_{b,y}}{\rho_2} \quad (1f) \\
\frac{\partial b}{\partial t} &= 0, \quad (1g)
\end{align*}
\]

where \( i = 1 \) denotes the upper freshwater layer and \( i = 2 \) the lower saltwater layer as introduced above. The model solves for the longitudinal and transversal components of discharge for unit width, \( q_{1x} \) and \( q_{1y} \), respectively, in both the layers, the elevation \( H_1 \) of the free surface and the elevation \( H_2 \) of the interface surface. Each layer is characterized by a constant density \( \rho_i \), already defined. The interface shear stress \( \tau_{int} \) and the bed shear stress \( \tau_b \) are computed as follows:

\[
\begin{align*}
\tau_{int,x} &= \rho_1 f_{int} \left( u_{1x} - u_{2x} \right) + \left( u_{1y} - u_{2y} \right)^2 \\
\tau_{int,y} &= \rho_2 f_{int} \left( u_{2x}^2 + u_{2y}^2 \right) \quad (2a)
\end{align*}
\]

where \( u_{1x} = q_{1x}/(H_1 - H_2) \) and \( u_{1y} = q_{1y}/(H_1 - H_2) \) are the longitudinal and transversal components of the velocity in the upper layer, and \( u_{2x} = q_{2x}/(H_2 - b) \) and \( u_{2y} = q_{2y}/(H_2 - b) \) are the velocity components in the lower layer. Moreover, \( f_{int} \) and \( f_b \) are the interface and bottom friction coefficients, respectively.

An explicit FORCE'-Contact scheme, based on Lax–Wendroff and Lax–Friedrichs schemes, is used to solve numerically the system of partial differential equations by using a finite volume technique (Canestrelli & Toro 2012; Canestrelli et al. 2012). The scheme is shock-capturing and, hence, allows to solve the possible internal discontinuities of the solution (e.g. hydraulic jumps). The domain is discretized by a triangular unstructured grid. Time marching is ruled by the Courant–Friedrichs–Lewy condition, which ensures the stability of the numerical scheme.

The weir structure is a physical barrier that divides the flow domain in two parts, upstream and downstream of it. This structure affects flow characteristics in space and time. The structure can be modelled in different ways. One possibility would be to refine the mesh in correspondence to the structure, such that the latter is fully resolved. This approach implies the use of a series of very small cells when the barrage thickness is small, and this would limit the maximum time step required for numerical stability.

Figure 5. Scheme of the approach for weirs. The structure is made coincident to the edges of the mesh and the computational domain is physically cut in the barrage section. The two new formed boundaries are internal boundaries where discharges and surfaces elevations depend on each other due to the barrage presence, indicated by the arrow.
In this work we thus use another approach, outlined in Figure 5. The upstream and downstream edges of the structure are taken to coincide with the edges of the upstream and downstream mesh. The computational domain is thus physically cut at the barrage section and split in two parts (Defina 1997).

The two new formed boundaries (just upstream and downstream of the weir) are internal boundaries that divide the flow domain in two sub-domains. A numerical procedure then computes the hydraulic conditions (i.e. discharges and surfaces elevations) at the two internal boundaries. These conditions may in general be assigned to cell nodes or edges of the computational grid cells, depending on the adopted numerical scheme. In the following these conditions are explained for a weir crossed by either a one- or two-layer flow.

4. Numerical procedure for weirs

In general, under unsteady flow conditions the presence of a barrage in a single layer flow may yield three different hydraulic conditions: disjointed flow sub-domains, partially linked flow sub-domains or totally linked flow sub-domains. In the first case there is no flow overpassing the structure; therefore the upstream and downstream sub-domains are completely independent; in the second case the flow is influenced only from upstream; in the third case the flow is drowned, and both upstream and downstream conditions affect the solution at the barrage location.

The situation is a little more complex if the flow field is stratified. The present aim is to obtain a numerical procedure that can be easily implemented to model the various flow conditions. The flow is assumed to be perpendicular to the weir axis and to vary slowly in time; streamline curvatures are assumed to be negligible. Such a flow field can be modelled by a one-dimensional steady flow characterized by a hydrostatic pressure distributed along the vertical direction. Note that we also assume that the interface between the two layers keeps sharp also when crossing the obstacle and, hence, the layers can be considered independent from each other. Finally, every weir of the barrage is assumed to be independent from the others.

The relevant flow variables are the discharge \( Q_1 \) and the surface elevation \( H_1 \) of the \( i \)th layer. The flow direction at the weir top depends on elevations of freshwater surface and of saltwater interface at the two sides. Here, with reference to the considered \( i \)th layer, the side having a higher upper surface is denoted as the upstream side (subscript ‘\( u \)’), while the other side is denoted as the downstream side (subscript ‘\( d \)’). Each layer is thus characterized by its own upstream and downstream sides. At generic time step the discharge of the \( i \)th layer depends on quantities calculated at the previous step:

\[
Q_1^{(n+1)} = f [Q_i, H_{iu}, H_{id}, H_{ju}, H_{jd}],
\]

where \( j \neq i \) denotes the other layer, while the superscript \( (n+1) \) denotes the new time steps. For the sake of simplicity, the various quantities, unless otherwise specified, refer to the generic time step \( n \).

A given weir of the barrage is characterized by three time-independent parameters: the crest elevation \( h_{cr} \), the crest width \( B \) and the discharge coefficient \( C_q \). The latter may assume constant or depth-dependent values, obtained from laboratory models or from literature. The widths \( B_u \) and \( B_d \) refer to the upstream and downstream sides of the crest, respectively. The hydraulic behaviour of the weir depends on the number of involved layers, that is, one or two, and is in general controlled by the elevations of the freshwater surface and of the saltwater interface at the two sides of the structure. In order to analyse the various flow conditions, we consider a straight frictionless channel, infinitely large and of infinite length under steady flow conditions, in which a weir is placed through the cross-section. The procedure is explained in detail for both a one-and two-layer flow field, but it can be easily extended to an \( n \)-layer flow field.

4.1. One layer frictionless straight channel

The weir is crossed only by the upper freshwater layer if the saltwater interface does not reach the crest either upstream or downstream of the weir. Clearly, if also the free freshwater surface does not exceed the crest elevation, no flow occurs and the two weir sides are hydraulically disjointed.

The critical flow depth above the crest is, by definition:

\[
y_{1c} = \frac{\sqrt{Q_1^2}}{gB^2}
\]

where \( g \) is the gravity acceleration. Two different cases, depending on free surface elevation and the value of the critical depth (4) upon the crest, can occur.

If \( H_{1d} < h_{cr} + y_{1c} \), critical flow conditions establish on the crest. The flow crossing the structure depends only on the upstream free surface elevation (Figure 6(a) and 6(b)). The freshwater discharge is:

\[
Q_1^{(n+1)} = C_q B \sqrt{2g} (H_{iu} - h_{cr})^{3/2}.
\]

The flow becomes supercritical at the downstream side, and a hydraulic jump forms where the momentum of the supercritical flow balances the momentum of the subcritical flow. An energy balance between the weir section and a downstream section located before the hydraulic jump gives the specific energy of the upper downstream layer with respect to the interface surface:

\[
E_{1d} = h_{cr} + \frac{3}{2} y_{1c} - H_{2d}.
\]

This energy value is inserted into an iterative formula to calculate the supercritical flow depth (subscript, ‘\( s \)’) at
the conjugate depth has to be compared with the depth downstream of the weir is computed as follows:

\[ y_{1ds}^{(n+1)} = \frac{Q_{1}^{(n+1)}}{B_d[2g(E_{1d} - y_{1ds}^{(n+1)})]^{1/2}}. \]  

(7)

The downstream supercritical depth (7) is then used to calculate the conjugate depth associated with the hydraulic jump (subscript, ‘c’):

\[ y_{1dc}^{(n+1)} = \frac{1}{2}y_{1ds}^{(n+1)} \left[ 1 + [1 + 8(F_{1ds}^{(n+1)})^{1/2}] \right], \]  

(8)

where \( F_{1ds}^{(n+1)} \) is the Froude number of the downstream supercritical flow just before the hydraulic jump. The conjugate depth has to be compared with the depth downstream to the jump, \( y_{1d} = H_{1d} - H_{2d} \). If \( y_{1d} < y_{1dc}^{(n+1)} \), the momentum upstream of the hydraulic jump is larger than the downstream momentum (Figure 6(a)) and the hydraulic jump is pushed in the downstream direction. As a consequence, the free surface elevation just downstream of the weir is computed as follows:

\[ H_{1d}^{(n+1)} = H_{2d} + y_{1ds}^{(n+1)}. \]  

(9)

Otherwise, if \( y_{1d} \geq y_{1dc}^{(n+1)} \), the hydraulic jump is pushed against the weir and no elevation has to be computed for the downstream weir side (Figure 6(b)).

If \( H_{1d} \geq h_{cr} + y_{1c} \) the structure is drowned and the flow depends on both the upstream and downstream free surface elevations (Figure 7).

An energy balance between two sections close to the weir, one upstream and the other downstream, reads:

\[ E_u - \Delta E = E_d, \]  

(10)

where the continuous friction losses have been neglected owing to the short length of the considered reach. The first and the last terms in Equation (10) are the total energy of the flow at the considered sections:

\[ E_u = E_{1u} = H_{1u} + \frac{Q_{1}^{(n+1)}}{2gB_{1}^{2}(H_{1u} - H_{2u})^{2}}, \]  

(11a)

\[ E_d = E_{1d} = H_{1d} + \frac{Q_{1}^{(n+1)}}{2gB_{1}^{2}(H_{1d} - H_{2d})^{2}}. \]  

(11b)

The term \( \Delta E \) accounts for energy losses due to the flow convergence before the obstacle and subsequent widening after it:

\[ \Delta E = (k_i + k_o) \cdot \frac{Q_{1}^{(n+1)}}{2gB_{1}^{2}(H_{1u} - H_{1d})^{2}}, \]  

(12)

where \( k_i \) and \( k_o \) are values less than 1 that depend on the shape of the weir wall. The energy balance (10) allows to compute the water discharge, which reads:

\[ Q_{1}^{(n+1)} = A\sqrt{2g(H_{1u} - H_{1d})} \]  

(13)

where \( A \) is a dimensional coefficient (m²) which depends on surface elevations at the previous time step:

\[ A = \left\{ 1 - \frac{1}{B_{1}^{2}(H_{1d} - H_{2d})^{2}} + \frac{k_i + k_o}{B_{2}^{2}(H_{1u} - h_{cr})^{2}} - \frac{1}{B_{2}^{2}(H_{1u} - H_{2u})^{2}} \right\}^{-1/2}. \]  

(14)
The discharge in the lower layer is zero, as assumed at the beginning of this subsection.

4.2. Two-layer frictionless straight channel

When the saltwater interface reaches the crest, the weir is overtopped by the flow taking place in both layers. For the lower saltwater layer, similarly to Section 4.1 two different cases can occur.

If \( H_{2d} < h_{cr} + y_{2c} \), a critical depth forms on the weir (Figure 8(a)) and the lower discharge reads:

\[
Q_2^{(n+1)} = C_q B \sqrt{2g(H_{2u} - h_{cr})^{3/2}}. \tag{15}
\]

The iterative formula for the supercritical depth (subscript, ‘s’) downstream to the weir reads:

\[
y_{2d,s}^{(n+1)} = \frac{Q_2^{(n+1)}}{B_d[2g(E_{2d} - y_{2d,s}^{(n+1)} - r_p(H_{1d} - H_{2d}))]} \tag{16}
\]

where \( r_p = \rho_1/\rho_2 \) is the density ratio. The specific energy reads:

\[
E_{2d} = h_{cr} + \frac{3}{2} y_{2c} + r_p (H_{1u} - H_{2u}) - z_d. \tag{17}
\]

As described above, the conjugate depth \( y_{2d,c}^{(n+1)} \) downstream of the jump must be calculated and compared to the downstream depth \( y_{2d} = H_{2d} - z_d \). If \( y_{2d} < y_{2d,c}^{(n+1)} \) the supercritical momentum is larger than the downstream subcritical momentum (Figure 8(a)) and the hydraulic jump is pushed downstream. The interface elevation just downstream of the weir is assigned as follows:

\[
H_{2d}^{(n+1)} = z_d + y_{2d,c}^{(n+1)}. \tag{18}
\]

Otherwise, if \( y_{2d} \geq y_{2d,c}^{(n+1)} \) the hydraulic jump moves close to the structure and no elevation has to be assigned at the downstream weir side (Figure 8(b)).

If \( H_{2d} \geq h_{cr} + y_{2c} \) the saltwater flow crossing the structure is drowned and depends on the interface elevations both upstream and downstream of the weir (Figure 9).

The energy at both weir sides and the localized energy dissipation now read:

\[
E_u = E_{2u} = H_{2u} + \frac{(Q_2^2)^{(n+1)}}{2gB_2^2(H_{2u} - z_u)^2} + r_p (H_{1u} - H_{2u}), \tag{19}
\]

\[
E_d = E_{2d} = H_{2d} + \frac{(Q_2^2)^{(n+1)}}{2gB_2^2(H_{2d} - z_d)^2} + r_p (H_{1d} - H_{2d}), \tag{20}
\]

\[
\Delta E = (k_i + k_o) \cdot \frac{(Q_2^2)^{(n+1)}}{2gB_2^2(H_{2u} - h_{cr})^2}. \tag{21}
\]

Energy balance finally leads to the following discharge formula:

\[
Q_2^{(n+1)} = A \sqrt{2g[H_{2u} - H_{2d} + r_p(H_{1u} - H_{2u} - H_{1d} + H_{2d})]}, \tag{22}
\]

where \( A \) (m²) depends on the saltwater interface elevations computed at the previous time step.
Figure 10. Weir crossed by both lower and upper layers. The flows may be directed in the same direction (a) or have opposite directions (b). In the first case, the upper-layer flow crossing the weir can be critical or drowned. In the second case, the upper-layer flow is drowned; therefore the crossing discharge depends on the surface elevations both upstream and downstream of the weir.

\[
A = \left\{ \frac{1}{B_2^2(H_2 - z_2)^2} + \frac{k_i + k_o}{B_2^2(H_2 - h_{cr})^2} \right\}^{-1/2}.
\]  

(23)

In all the cases treated so far, the weir is completely immersed in the lower saltwater layer. Therefore, the upper layer is not directly affected by the presence of the weir. As a consequence, the elevation \(H_{2,cr}\) of the interface at the crest replaces \(h_{cr}\). In addition to the flow direction of the upper layer, also the relative flow direction between the layers has to be considered.

If the flows in the two layers have the same directions (Figure 10(a)), the possible flow conditions are as explained in Section 4.1. Otherwise, if the flows are in opposite directions (Figure 10(b)), the upper layer discharge is computed from a total energy balance upstream and downstream of the weir neglecting continuous energy losses (owing to the short length of the considered reach):

\[
Q^{(n+1)}_1 = A\sqrt{2g(H_{1u} - H_{1d})},
\]

(24)

where \(A\) (m²) depends on the elevations of saltwater interface and freshwater surface at the previous time step:

\[
A = \left\{ \frac{1}{B_2^2(H_1 - H_2)^2} - \frac{1}{B_2^2(H_1 - H_2)^2} \right\}^{-1/2}.
\]

(25)

5. Application to the Adige River: simulations and results

The model has been calibrated to reproduce the thickness of the freshwater and saltwater layers in the positions surveyed during the field measurements carried out in

Figure 11. Computational domain includes the final reach of Adige River and a relatively little part of the Adriatic Sea. The length of the considered river reach is about 13 km, with a width varying between 130 m and 860 m. The arrow points the position of the submerged barrage built to contrast the seawater intrusion.

Figure 12. Elevation of the saltwater interface (red line) and the free freshwater surface (blue line) at a point located just upstream of the barrage, for both computational configurations. A refers to the case in which the barrage weirs are modelled with triangular cells having the same elevation of the weirs crests; B refers to the case in which the computational domain is split by the barrage in two sub-domains linked together by the hydraulic conditions described in Section 4.
August 2012. The friction coefficients resulting from this calibration are $f_{int} = 0.001$ for shear stress at the interface between saltwater and freshwater (Equation 2a) and $f_b = 0.01$ for the bed shear stress (Equation 2b).

The simulated period lasts 108 hours. The computational domain, shown in Figure 11, includes the final reach of the Adige River and a relatively small part of the Adriatic Sea. The considered river reach is about 13 km long and has a width varying between 130 and 860 m. A constant freshwater discharge of 70 m$^3$/s is assigned at the upstream boundary, on the basis of the measured freshwater data (Figure 2(a)). Tidal elevations have been assigned at the offshore boundary (Figure 2(b)) by considering the measurements made during the salinity monitoring campaign.

Two different computational configurations have been investigated. In the first (denoted with the letter A) the seven weirs of the barrage gates are modelled directly with small triangular cells having the same elevation of the weir crests. The numerical finite volume model is thus applied to the entire computational domain without any internal boundary. The second computational configuration (denoted with letter B) models the behaviour of the weir barrage through the hydraulic relations linking together the two sub-domains identified by the barrage itself, as described in the previous sections. A constant discharge coefficient, $C_q = 0.40$, has been assumed for the weirs when critical conditions occur above the crest.

Figure 12 shows, for both computational configurations, the elevation of the saltwater interface and the free freshwater surface at a point located just upstream of the structure. Simulation of type A provides slightly lower elevations for both surfaces, the difference being in the order of 10 cm. In any case, numerical simulations show that the saltwater interface is in-phase with the freshwater surface and, hence, its behaviour strongly depends on the astronomical tide of Adriatic Sea, as it was obvious to expect. Figure 13 shows the longitudinal profiles of the saltwater interface and free surface elevations at the channel axis for low (Figure 13(a)) and high tide (Figure 13(b)) conditions. The freshwater flow in the Adige River is from left to right, and the origin of the horizontal axis is at the river mouth.

Tide affects the elevation of the saltwater interface, the seawater intrusion length being about 9 km. However, independently of the tidal phase, the extension of seawater intrusion is about 800 m shorter when the barrage weirs are discretized through small triangular cells and a non-split computational domain is considered (configuration A). On the other hand, the interface elevation evaluated by dividing the computational domain in two sub-domains, linked by suitable hydraulic conditions (configuration B) attains lower values (with respect to case A) downstream of the barrage (right side of Figures 13(a) and 13(b)), whereas it is higher upstream (left side, in figures). It can then be concluded that modelling the barrage weirs by simply adapting the dimensions and the shape of the grid cells (case A) tends to underestimate salt wedge elevations in the reach in which saltwater effects are more dangerous, that is, upstream of the barrage, if compared to the results obtained considering a split computational domain B.

6. Conclusion

A finite volume shock-capturing model has been used to solve the two-dimensional, two-layer shallow water equations that govern the salt wedge dynamics. The model has been used to investigate the seawater intrusion into the estuary of the Adige River (Italy). The tidal range at the river mouth and the discharge regime are such that a sharply stratified salt wedge intrusion takes place along the river estuary during the summer period. In order to model the submerged barrage built to prevent seawater intrusion, the computational domain has been built by either reducing the size of the computational grid cells in the correspondence of the barrage or treating the barrage weirs as an internal boundary. In this latter case, suitable hydraulic
conditions have been applied to link the relevant variables upstream and downstream of the barrage. These conditions have been implemented in the model and are found to manage correctly and efficiently the behaviour of barrage weirs, for both one- and two-layer conditions. The procedure can be easily extended to n-layers as well as to other structures that introduce a physical discontinuity into the flow domain (e.g. weirs, spillways, gates, bottom openings).

The numerical results relative to the configuration discretizing the barrage weirs by adapting the cell size tend to underestimate elevations of both the saltwater interface and the free freshwater surface. In this case a shorter seawater intrusion is predicted with respect to the case in which the computational domain is split in two sub-domains, linked together by the relevant hydraulic relations described in Section 4. However, both simulation types confirm that the barrage does not work efficiently, as mentioned in the introduction.

The barrage is always installed during summer (except for maintenance, which anyway involves one weir per time); therefore salinity measurements without it are not available. In addition, modelling the seawater intrusion without the barrage would provide not physically-based results, because the morphology of the river bed has been modified by the barrage presence itself over the years. On the basis of the preceding considerations, we deem that, from an engineering point of view, the approach with internal boundaries is much more conservative, such that it tends to predict a larger salt intrusion and, hence, it ensures higher safety standards for an improvement of the structure.

Treating weirs through appropriate hydraulic relations instead of considering them as elevations of the river bed leads to an improvement of the computational efficiency, due to the minor refinement of the mesh. In this latter case, moreover, integration time step can be increased preserving anyway the Courant–Friedrichs–Lewy condition required for the stability of the numerical scheme.

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No potential conflict of interest was reported by the authors.

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